Domain-size control by global feedback in bistable systems

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We study domain structures in bistable systems such as the Ginzburg-Landau equation. The size of domains can be controlled by a global negative feedback. The domain-size control is applied for a localized spiral pattern.

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Spatially localized states have been observed experimentally in binary fluid mixtures [1], in electroconvection in nematic liquid crystals [2], and in granular media undergoing the Faraday instability [3]. Some simple model equations have been studied to understand the mechanism of the localized states found in dissipative systems. Solitonlike localized states have been found in the quintic complex Ginzburg-Landau equation and the coupled complex Ginzburg-Landau equations [4,5]. Wormlike localized states were studied with the anisotropic complex Ginzburg-Landau equation coupled with a scalar mode [6]. A self-trapping mechanism works for localized states in the quintic Swift-Hohenberg equation [7]. Long-range inhibition is important for localized states in some reaction diffusion equations [8]. On the other hand, controlling chaotic dynamics has been investigated with the Ott-Grebugi-Yorke method and the feedback method [9,10]. Zykov *et al.* studied the control of spiral waves in a spatially extended system by global feedback [11].

We study the control of the domain size of localized domains in spatially extended bistable systems. Our first model equation is based on the Ginzburg-Landau equation coupled with an inhibitory medium:

$$\frac{\partial u}{\partial t}=u-u^3-v+\frac{\partial^2 u}{\partial x^2},$$

$$\tau\frac{\partial v}{\partial t}=-v+K(u-c)+D\frac{\partial^2 v}{\partial x^2}, \quad (1)$$

where u obeys the Ginzburg-Landau equation, v denotes the inhibitory variable, D is the diffusion constant of v, τ is the time constant of v, and c is a parameter between -1 and 1. This model equation is a reaction diffusion equation. The system size is L and the periodic boundary condition or the no-flux boundary condition is assumed. If D is sufficiently large, a localized state is obtained as in [8]. If D is infinitely large, v is uniform for $0<x<L$, that is, $v=\langle v\rangle=(1/L)\int_0^L v\,dx$. The second equation in Eq. (1) is reduced to

$$\tau\frac{d\langle v\rangle}{dt}=-\langle v\rangle+K(\langle u\rangle-c), \quad (2)$$

where $\langle u\rangle=(1/L)\int_0^L u\,dx$. If the adiabatic approximation is assumed, $\langle v\rangle=K(\langle u\rangle-c)$. Substitution of this relation into the first equation of Eq. (1) yields

$$\frac{\partial u}{\partial t}=u-u^3-K(\langle u\rangle-c)+\frac{\partial^2 u}{\partial x^2}, \quad (3)$$

where the third term on the right-hand side represents the global negative feedback. If the third term $-K(\langle u\rangle-c)$ takes a small constant value b, there are two stable states near $u\sim\pm 1$. If the initial condition takes a domain structure, the domain wall moves with a constant velocity for nonzero b. The domains of positive (or negative) u become dominant if b is positive (or negative). In our negative feedback model with $K>0$, the effective control parameter $-K(\langle u\rangle-c)$ decreases (increases) as the domain size of positive (negative) u increases. Finally, $\langle u\rangle=c$ is attained and the domain growth stops. Then, the size of the domain of $u=\pm 1$ is approximately $(1\pm c)L/2$, namely, the parameter c determines the domain size. We can control the domain size by changing the parameter c. Figures 1(a) and 1(b) display the

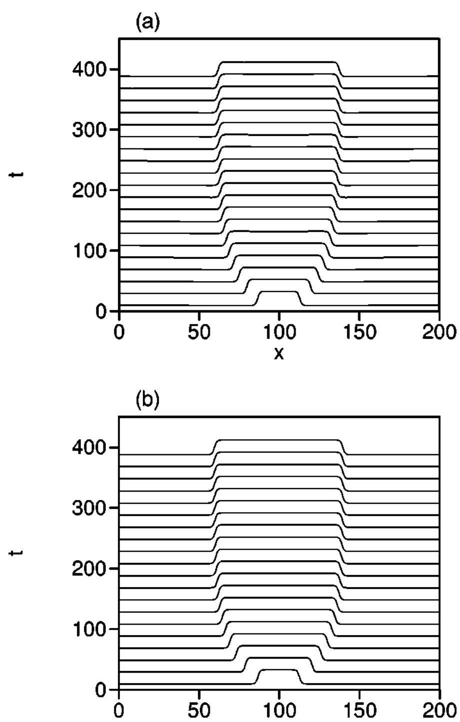

FIG. 1. (a) Time evolution of $u(x,t)$ by Eq. (1) at $\tau=1$, $K=0.5$, $c=-0.2$, $D=10\,000$, and $L=200$. (b) Time evolution of $u(x,t)$ by Eq. (3) at $K=0.5$, $c=-0.2$, and $L=200$.

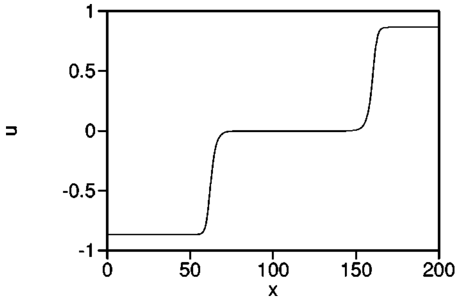

FIG. 2. Final stationary state for Eq. (5) at $K_1=K_2=0.5$, $c_1=-0.1$, $c_2=0$, and $L=200$.

time evolution of the domain structure at $K=0.5$, $c=-0.2$, and $L=200$. The initial condition is $u(x)=1$ for $98<x<102$ and $u=-1$ for $x<98$ and $x>102$. The numerical simulation was performed with the pseudospectral method of mode number 1024 and time step 0.01. Figure 1(a) displays the time evolution for Eq. (1) with $D=10\,000$ and $\tau=1$. A localized state with size 75.7 is obtained. The domain size is calculated as the width of the region of $u>0$. Figure 1(b) displays the time evolution by the Ginzburg-Landau equation (3) with global negative feedback. A domain with a fixed size is obtained as a stationary state. The final size of the domain $u=1$ is 80. Since the diffusion constant $D=10\,000$ in Eq. (1) is very large, the time evolution obtained by Eq. (1) is close to that of Eq. (3). The domain size is well approximated at the value $(1+c)L/2=80$.

We can control the domain size even if the system has three stable states. The model equation is the quintic Ginzburg-Landau equation:

$$\frac{\partial u}{\partial t}=-au+u^3-u^5+e+\frac{\partial^2 u}{\partial x^2}, \tag{4}$$

where a and e are parameters. If $e=0$ and $a=3/16$, the potential energy $U=au^2/2-u^4/4+u^6/6-eu$ takes the same local minimum value 0 at $u=0$ and $\pm\sqrt{3}/2$. Two kinds of domain wall, which connect $-\sqrt{3}/2$ and 0 and 0 and $\sqrt{3}/2$, do not move at the parameters $e=0$ and $a=3/16$. For the other parameter values, the domain walls have finite velocities. We assume a model with global negative feedback as

$$\frac{\partial u}{\partial t}=u^3-u^5-K_1(\langle u\rangle-c_1)-K_2(\langle u^2\rangle-c_2)u+\frac{\partial^2 u}{\partial x^2}, \tag{5}$$

where $\langle u\rangle=(1/L)\int_0^L u\,dx$, $\langle u^2\rangle=(1/L)\int_0^L u^2 dx$, and K_1,K_2 are coupling constants and c_1,c_2 are the control parameters that determine the domain sizes. Since the parameter value $(a,e)=(0,0)$ in Eq. (4) is a codimension-2 point, we need two types of negative feedback. If the negative feedback succeeds, the final state satisfies $\langle u\rangle=c_1$ and $K_2(\langle u^2\rangle-c_2)=3/16$. We have performed a numerical simulation for $L=200$ under the no-flux boundary condition. The initial condition is $u(x)=-1+2x/L$. Figure 2 displays the final state in the time evolution by Eq. (5) at $K_1=0.5$, $K_2=0.5$, $c_1=-0.1$, and $c_2=0$. There appear three domains of $u\sim-\sqrt{3}/2$, 0, and $\sqrt{3}/2$. If the domain sizes are denoted as l_+, l_0, and l_-, respectively, for the three domains of $u=\sqrt{3}/2$, 0, and $-\sqrt{3}/2$, the domain sizes satisfy approximately $\langle u\rangle=\sqrt{3}/2(l_+-l_-)/L=c_1=-0.1$ and $\langle u^2\rangle=3/4(l_++l_-)/L=3/(16K_2)+c_2=3/8$. The expected domain sizes are $l_+=38.4$, $l_0=100$, and $l_-=61.6$ for the parameters $c_1=-0.1$, $c_2=0$, and $L=200$. The numerical result is approximately $l_+\sim39.7$, $l_0\sim97.5$, and $l_-\sim62.7$. The global feedback succeeds in domain-size control for this quintic Ginzburg-Landau system.

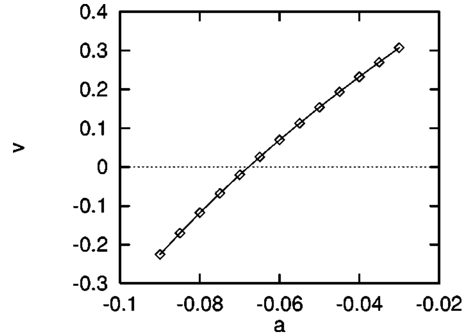

FIG. 3. Velocity v of the domain wall as a function of a for Eq. (6) at $c_2=0.4$.

The same method is applicable for nonvariational systems. We use the quintic complex Ginzburg-Landau equation:

$$\frac{\partial W}{\partial t}=-aW+(1+ic_2)|W|^2W-|W|^4W+\frac{\partial^2 W}{\partial x^2}, \tag{6}$$

where W is a complex variable and c_2 is a nonvariational parameter. There are two stable uniform states: $W=0$ and $W=W_0\exp(i\omega_0 t)$, where $W_0=\sqrt{(1+\sqrt{1-4a})/2}$ and $\omega_0=c_2(1+\sqrt{1-4a})/2$. There exists a domain wall that connects the zero state and the oscillating state. Figure 3 displays the numerically obtained velocity of the domain wall as a function of a for $c_2=0.4$. The positive velocity implies that the oscillating state invades the zero state. The velocity of the domain wall is 0 at $a=a_c\sim0.0678$. The model equation with global feedback is

$$\frac{\partial W}{\partial t}=-aW+(1+ic_2)|W|^2W-|W|^4W-K\langle|W|^2\rangle W+\frac{\partial^2 W}{\partial x^2}, \tag{7}$$

where K denotes the feedback strength and $\langle|W|^2\rangle=(1/L)\int_0^L|W|^2dx$. Figure 4 displays the time evolution of $|W|$ for $a=0.01$, $L=400$, $c_2=0.4$, and $K=0.12$. The initial condition is $\mathrm{Re}W(x)=1$ and $\mathrm{Im}W(x)=0$ for $196<x<204$, and $W(x)=0$ for $x<196$ and $x>204$. The domain size l of the oscillating state is calculated by the relation $a+K\langle|W|^2\rangle\sim a+K|W_1|^2l/L\sim a_c$, where $|W_1|\sim0.866$ is the amplitude of the oscillating state coexisting with the zero state at $a=a_c$ for Eq. (6). The estimated value is $l=L(a_c$

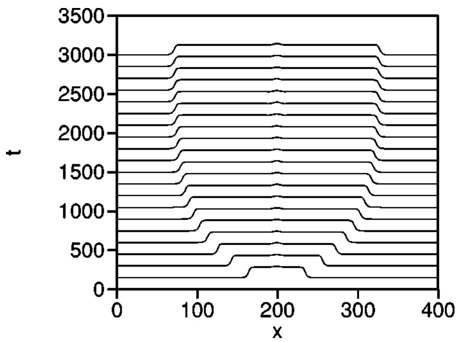

FIG. 4. Time evolution of $|W|$ for Eq. (7) at $a=0.01$, $c_2=0.4$, $K=0.12$, and $L=400$.

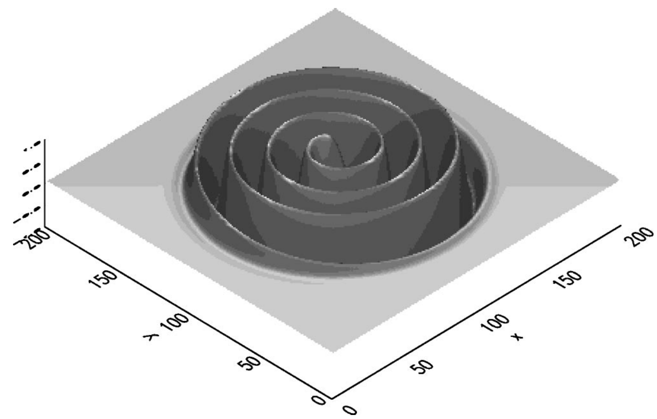

FIG. 5. Three-dimensional plot of ReW for a localized spiral pattern as a result of the time evolution via Eq. (8) at $a=-0.05$, $c_2=0.4$, $K=0.4$, and $L=200$.

$-a)/(K|W_1|^2)\sim 256.8$. The numerically obtained size is approximately 257. We can control the domain size by changing the parameter a or K.

We have performed a simulation of the two-dimensional quintic complex Ginzburg-Landau equation with global feedback:

$$\frac{\partial W}{\partial t}=-aW+(1+ic_2)|W|^2W-|W|^4W-K\langle|W|^2\rangle W+\nabla^2 W, \qquad (8)$$

where K denotes the feedback strength and $\langle|W|^2\rangle=(1/L^2)\int_0^L\int_0^L|W|^2dxdy$. The parameters are $L=200$, $a=-0.05$, $c_2=0.4$, and $K=0.4$. The numerical simulation was performed with the psudospectral method of mode number 256×256 and time step 0.005. The initial condition was Re$W(x,y)=0.0033(x-L/2)(r_d-r)$ and Im$W(x,y)=0.0033(y-L/2)(r_d-r)$ for $r<r_d$ where $r=\sqrt{(x-L/2)^2+(y-L/2)^2}$ and $r_d=55$, and $W=0$ for $r>r_d$. That is, a topological defect is set at the center $(x,y)=(L/2,L/2)$ as an initial condition. A spiral pattern evolves from the initial condition. The spiral pattern occupies only a finite domain because of global feedback. Figure 5 displays a three-dimensional plot of Re$W(x,y)$ for the localized spiral pattern. The value of $a+K\langle|W|^2\rangle$ is approximately -0.0648 for the stationary localized spiral, and the value is close to a_c, but slightly larger. This probably is due to the surface tension effect. Recently, a localized spiral pattern was studied with the quintic complex Ginzburg-Landau equation without the global feedback term [12]. However, the localization mechanism is different. In our model, the domain size of the spiral pattern can be controlled by the parameter a or K.

To summarize, we have studied the control of the domain size for Ginzburg-Landau type equations with global negative feedback. The domain-size control by global negative feedback is one of the simplest examples of control for spatially extended dynamical systems. The domain size is determined by the condition that the velocity of the domain wall is zero. The global negative feedback can be derived from the coupled reaction diffusion equation. In the crystal growth problem, the temperature field plays the role of the inhibitory medium through the latent heat released at solidification. The global negative feedback appears more naturally in electric circuits [13]. The control of the domain size is fairly robust and may be applicable for many systems.

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